Louis De Broglie’s experiment

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Abstract

Louis de Broglie’s celebrated hypothesis transfers a problem of representation of optics to the quantum theories. Let us suppose that the fact that originated the problem in optics is the following, “The images obtained by optical instruments are limited by diffraction. Is the diffraction phenomenon exclusively restricting of the information contained in the image or does it have some physical meaning by itself?” This considered as a problem of measurement of the radiative field does not pose the question of tracing well-defined processes in the sense of the light quantum concept. Despite this the application thereto of quantum formalism makes the essentially inferring statistical nature of measurement forecasting most important. We suggest avoiding the probabilistic conception of measurement by replacing the wave-corpuscle dualistic principle with the duality one. The latter is a basic feature of projective geometry. Referring to it we shall try to include the diffraction in a geometrical model of optics and interpret in that coherent schema the point-plane “wave” dual solution. We show what experimental substantiation the proposed duality has. Lastly, we again discuss the experimental basis of wave-corpuscle dualism.

1 Introduction

According to the literature [1] in the thesis work of L. de Broglie [2], ‘recherches sur la théorie des quanta’ the famous hypothesis that the same wave and corpuscle dualism as is present in light may also occur in matter is formulated. The quantum mechanics [3] accepts the correctness of the terms because it places them there where they are expounded in the manuals. Although it discusses the consequences, it does not seem to assess the assumptions. Let us consider two
aspects, one about the widespread use of calling it a hypothesis and the other about the undulatory and corpuscular representation of light.

Precisely it is written that de Broglie formulated a hypothesis. It concerns a metaphorical analogy of light with matter. We believe that, thus formulated, the hypothetical analogy renounces interpretation of the facts. We want to explain this. A mathematical hypothesis may be formulated within the framework of a unified theory of light and matter [4]. A theory which is not mechanics in short because this does not explain typically luminous effects if there are any. But in 1924 there is still no common theory of light and matter [5]. Therefore at that time the hypothesis is not a conjecture formulated within a theory. It might be a supposition on which to base a new research program. As it concerns physics it should rest on a natural phenomenon. But the hypothesis precedes in time all the experiments aimed at verifying it. Indeed, on the one hand it does not grasp any evident resemblance between light and matter. For example, the sun reflected by the burning glass mirror does not bombard ships exactly like a cannon ball. On the other hand the oft quoted scrupulous experiment of C. Davisson and L. Germer [6] is of 1927. This experiment confirms the de Broglie hypothesis in the sense that based on electronic diffraction measurements from a nickel crystal it associates a wavelength with the collimated electronic emission. This could be used to confute the conclusions reached by J.J. Thomson about the corpuscular nature of cathode rays just as well as it was used to prove the undulatory nature of matter. In short, in 1924 the hypothesis anticipates both the theory and the verification experiments.

The second imprecision of the literature concerning the contribution of de Broglie seems to us to consist of a reversal of the order in which the words appear. In our opinion the hypothesis would state more correctly, “the same dualism of wave and corpuscle as is present in matter may also occur in light”. Indeed, not only does it not deal with modeling mechanics in accordance with a dualistic theory of radiation already formalized, but in addition both the emissive material conception of I. Newton and the elastic one of A. Fresnel belong to dynamics so that exactly the dualist concept of light is mechanical. But in 1924 there are at least two formalisms foreign to the conventional mechanical schema, the electromagnetic theory and the quantized energetics of A. Einstein [7]. The first theory concerns more the propagation of electrical phenomena than the characterization of ether; the second more the energy balance of elementary processes [8] than corpuscularity. Since the principle of interference of Huygens-Fresnel can be fitted on electromagnetism [9] and in addition Einstein himself hastened to establish an equivalence between mass and energy [10], it may be thought to trace back the two attempts in the already broken-in tracks of mechanics [11]. This, which more than a hypothesis we would like to call an idea [12], is valid in the absence of reasons justifying the attempts to develop more original light theories.

In this work, which is not historical despite the problems faced being so, we shall briefly confront dualism in physics. Then we will seek to explain how geometry allows justification of the dual aspect of light phenomena in our opinion. In literature other possible geometrical interpretations [13] are discussed.
Lastly, we will seek to give a physical meaning to “de Broglie’s hypothesis” in accordance with the experimental results obtained earlier by E. Abbe [14]. This is the reason for the title we chose.

2 Dualism as a contribution to unification

De Broglie’s contribution to physics is to have contemplated two different entities, light and matter, and to have conceived that wave-corpuscle dualism can unite them [15]. This means that dualism is referred to undulatory or corpuscular pictorial descriptions but at the same time unifies the two entities light and matter.

In quantum mechanics the conflict between the wave and corpuscle concepts is resolved by postulating the principle of complementarity [16].

As to the essence of the two entities light and matter which the theory treats monistically, we see two possibilities. Either both exist in the outside world, which we as physicists admit, and dualism is accessible to the experiment. Or division of things in two (or more) classes naturally is not given and dualism between light and matter is equivalent to that between wave and corpuscle. Let us take the first case. If we consider the distinction between light elements and matter elements a natural fact, in physics we should discuss the experimental consistency of this not unifiable dualism between at least two types of distinct substances. Second case: but if we consider linking between experiences and pictures the work of physicists, dualism (or pluralism) takes place at the level of our understanding of the experiments; we are referring certain experiments to mechanics and we judge those remaining more linked to sensorial impressions. At this level, theories different from mechanics describe effectively different aspects of the external world. Since in addition dualism is not among the facts accessible to experimental investigation, neither does the level of its insertion among objects of the external world and their intellectual representation have an objective substantiation. In this case a dialectic comparison between alternative theories and conventions might not be contradictory.

To summarize, physics is the science of physicists but it is also a science which explains phenomena occurring in nature [17]. On de Broglie’s thesis it can be substantiated how he does not propose at all to put light manifestations in relation with material bodies. Rather, in the 1924 work he presumes that theories are platonic ideas and formulates his unifying hypothesis for two preexisting theories. He chooses analytical mechanics and geometrical optics. Geometrical optics is distinguished from analytical mechanics as long as the methods of the latter are not applied to the former. And this ensures the success of unification even if it perhaps makes superfluous formal verification of dualism separately for the light theory and for the matter theory.

Although it is innovative to introduce dualism in a theory, the dualistic aspect considered is not specific to light theories. To clarify what we mean, let us consider the contrasting pictorial descriptions which would unify the dynamics with optics. They are termed pictorial because they evoke in us familiar
concepts. Their conflict [10] precedes the formulation of de Broglie’s hypothesis because it expresses a contradiction between being and movement. The dilemma between the above mentioned concepts is a pillar of western philosophy and was fed already in ancient Greek philosophy by two schools, one termed Eleatic and the other Heraclitean. For the purposes of this article we agree that the first of the two terms, being, specifies an actual (“realised”) property on condition that it be permanently preserved. The mathematical magnitudes introduced are used to represent a state. The other term, movement or “becoming”, expresses a relationship between spatial positions of the same thing, for example along a trajectory, or indifferently between two or more things [18]. Mathematically if the relationships represented refer to pairs of facts they are termed binary, otherwise ternary et cetera. The variables represent not necessarily punctual relative localizations. The state of things cannot be part of the representation.

In classical mechanics, with being are associated the causes of motion represented mathematically by means of vectors or tensors (state of motion) while with becoming are associated displacements in geometrical space (kinematics). The data available allow establishing for each specific application which is the best adapted representation and therefore contradictions do not arise. To mix the two pictures it is necessary to abolish the conventional distinction between applied vectors (forces) and free vectors (displacements) which is what takes place by graphically representing both on the same sheet of paper.

Contrary to mechanics optics, as a theory, is contradictory. The great merit of the radiation theories of Einstein and Maxwell lies just in avoiding contradictoriness by directly applying a mathematics to a preselected class of light phenomena. In this work we shall consider the theory of the second because in the present state of telecommunications development it is applied to the signal. Einstein’s theory of radiation, since it puts on an equal footing modulation and statistical fluctuations, cannot include images like those of television when the television is tuned to a channel.

3 Contradictory pictures in optics

Since from the beginnings of logic the choice of basing a hypothetical-deductive theory on a dualistic principle appeared debatable, in the previous chapter we identified in pluralism a possible alternative to a well-founded theory. Optics is not a theory but a discipline which gathers together a large number of unrelated facts. The known facts, of which it is possible to specify the observation conditions, are truly many because a goodly part of our information about the external world is linked to the visual channel. The number of explanations given is also considerable. If we remain with the descriptions having objective corroboration, the first difficulty lies in disentangling the physiological from the physical contribution to visual impressions. The theories of colors are emblematic.
3 CONTRADICTORY PICTURES IN OPTICS

3.1 Graphic optics when diffraction is a small effect

If we go back to the history of optics, as does de Broglie in the thesis, we can observe that in the same period the two representations he calls “undulatory” (from C. Huygens’s; without periodicity principle) and “dynamic” (from Newton’s theory) are strictly relevant to the same experimental base and the opposition of ethereal wave-trains and luminous corpuscular matter takes place only at the interpretive level \[19\]. The physical content of optics commonly termed geometrical is summarized in graphic memoranda of the conditions of operation of well diaphragned instruments. These are used to view sharp images of known objects and are first of all the pinhole camera, then lenses, mirrors, telescopes and the like. The majority of the graphic memoranda analyze their operation in terms of reflection and refraction with recourse to the radial geometric element. Mathematically speaking the theories developed on rays are linear. In them the so-called optical aberrations are considered a small error which is allowed for by making corrections to the linear theory. If by applying differential calculus the graphic trace is corrected to higher orders, smooth curved lines replace the rays.

In the same period as Newton, Huygens was able to trace the same stigmatic images more elaborately by using a surface element associated with the ray-optical path, instead of the ray itself. He showed us that by assuming fronts with some extension instead of single trajectories it is possible to justify graphically even two distinct stigmatic images if the fronts have two different envelopes. Strictly speaking we could not call stigmatic a split image and would judge unusable for the purposes of observation an optical instrument which splits the objects observed. But Iceland spar appropriately cut and finished optically shows better than other crystals that it can form a neat double image. Let us describe the fact. Resting the crystal on a sheet on which a point is marked, the image thereof appears double and, rotating the spar, the duplicate executes one rotation around the ordinarily refracted point, on a visibly higher plane.

Huygens had believed that the split image was not explainable otherwise than by also assuming the propagation of non-spherical wave fronts inside the spar. Newton objected that it was a matter of understanding if one should assign the responsible property of the splitting of the image to the light matter or to an ethereal fluid. In the second case he didn’t understand why ether should possess some anisotropy inside the spar and not show any in intergalactic space. Actually the polarization could be understood in Newtonian terms and it was thus that E. Malus fixed it for posterity.

Again according to history, Newtonian optics did not reveal itself insufficient to explain the birefringence mentioned above but failed on the diffraction experiments of Fresnel.

Diffraction as a phenomenon had already been accurately described by Newton. But no one believed that, when seen under high magnification, it might prove to be a remarkable effect. In addition, the supposed inadequacy of geometrical optics to account for Fresnel’s experiments has been used more for setting precise limits of validity of the graphic analysis handed down than for
unequivocally identifying the fact to which to attribute the fanlike opening of
a diaphragmed light beam. Dispersion, interference, diffraction, diffusion... all
contribute to blurring the image. Although, contrary to Newton, we hesitate
to assign each as a separate property to light, nevertheless, if we want to allow
for the opening of the beam like a fan we must choose to deal with one of these
phenomena.

We are inclined to choose diffraction under the conditions of J. Fraunhofer,
first of all because the experimental conditions are well defined and then because
this diffraction can be treated mathematically. Ours is a choice and as such is
arbitrary. R.W. Hamilton made another choice. He found a mathematical
relation between a normal congruence of rays and a family of surfaces \[20\].
He also showed that integration of the differential equations of dynamics is in
relation with the integration of a first order partial differential equation (the
one for the generating function). The above relation can receive any convenient
physical interpretation \[21\] provided it is remembered that the phase space
and that of graphical optics are not isomorphous \[22\]. Hamilton applied
the mathematics developed by himself to the conical refraction of biaxial crystals.

Conical refraction is so called because the image of a point observed through
a biaxial crystal, instead of appearing split appears widened like the base of a
cone. Fraunhofer’s diffraction takes place under quite different conditions, i.e.
even in the absence of an interposed dielectric material. For this reason we
speak of a choice.

4 The problem of what moves in optics

Until now we have said that behind the early theories of light there are instru-
mentation manufacturing technologies which pursue magnification on condition
that images shall be similar to the original in form and colors. Even then opti-
cians were pragmatic, they assumed that the graphic prescription which allows
analysis of the image more conveniently was the most natural in a teleological
sense. Since Newton left us a (reflector) telescope they considered it certain
that he had understood the nature of light better than Huygens.

As concerns the Newtonian interpretation of the rays as particle trajectories
it is perfectly useless. It is not useful for improving the practice of processing
optical glasses of refractory telescopes. It is not even useful for further charac-
terizing by experiments the material nature of light; the proof of this lies in the
convincing explanations of diffraction based on the periodicity principle due to
T. Young and Fresnel.

Maxwell’ starting point is not oriented toward improvement of instrumen-
tation but, in modern terms, towards coherent representation of the signal received
when we can disregard instrumentation particularly. Not in judge’s clothing but
in a lawyer’s, we insert a digression to give an idea of the theoretical concep-
tion. Maxwell, contrary to Einstein, considers only the phenomenology linked
to movement. Consequently he does not characterize and does not interpret the
moving object which we are calling a signal (to distinguish it from a tangible
body). A signal can be received through being propagated along an electrical or optical transmission line. But if it is irradiated in space it can be detected mechanically, chemically, electrically, magnetically, thermally or optically. Since characterization of the signal is not the purpose of the theory, the latter is the more general the more extended is the class of signals considered. This generality cannot be unraveled as long as the specific job attributed to optics is to explain the transition of the object to the image. But this transition, once separated from the process of production and detection of the signal, is a sui generis movement. To the touch it is established that an object standing in the laboratory and visible to the naked eye does not move due to the fact that it is observed directly rather than with the aid of an optical instrument. And yet the image of the object in toto is formed visibly elsewhere. Either microscopic particles transit from the position of the object to that of the image, a case in which mechanics can be applied as is to light (theory of F. Hasenoehrl [23]) or the light can involve matter more ineffably. In the second case for the sake of argument the movement has no equivalent in classical kinematics.

Now let us go back to Maxwell. He conceives interference as proof of the immateriality of light. He reasons more or less thus: the description of fringes, requiring evaluation locally of the algebraic sum of two quantities – those which interfere – testifies against the emissive theories, at least as long as mass is the substantial characteristic. Indeed, usually to mass is attributed an essentially positive magnitude so that, contrarily to the electric charge, there is no analogy of Faraday’s cage capable of screening it. Following M. Faraday further, he lets himself be guided by spatial considerations; phenomenology develops entirely in our world and therefore describing it has sense at first. At that time, this might have seemed a foregone choice [24] [25].

A mechanical illustration is also part of Maxwell’s original interpretation. To make plausible the distribution of light intensity on the immaterial fringes, he identifies electromagnetic ether with luminiferous ether. The latter is a hypothesis on the indirect role of a medium during transmission; ether is a medium which is subject to elastic vibrations while light traverses it and therefore transmits the dynamic energy of light along definite directions. To dynamically characterize the propagation of light it is necessary and sufficient to specify the state of motion of ether in accordance with the theory of elasticity. Of course the dynamic characterization could continue to reveal itself perfectly useless in applications.

After Maxwell it was noticed that, whatever ether is, contradictory material properties should be attributed to it. Therefore it was preferred not to consider it a real entity. It follows that vacuum is not an entity replacing ether but is a non-entity. Denying the existence of electromagnetic ether and replacing it with nothing, nothing takes on a state of motion. In other words, an elastic vibration energy associated with light propagation causing the appearance of the fringes is lacking. Nor does thermal conduction account for the light distribution observed experimentally in a manner which might be consistent with electromagnetism in a vacuum. Indeed, the thermodynamic interpretation of the flux of light energy assumes that the thermodynamic state variables are functions of the spatial coordinates and possibly of time. It takes on this dependence to handle the
diffusion of heat by conduction from the hotter zones to the cooler zones of a body. But the coordinates describe the thermal state of the conductor. Part of the above discussion aims at conceptually identifying what might move while conveying energy in the absence of an interposed medium. In electromagnetism every general solution (without boundary conditions) represents a movement of permanent type. Although these movements have been interpreted mechanically as waves of ether, they do not have a mechanical characterization. For example, they have nothing in common with surface waves like those of the sea nor with wave fronts like those generated in a pan by emersion of a cork nor with the vibrations of tight ropes. They represent a movement dissociated from any other characterization. Maxwell explains this in the Theory of Heat, especially in chapters XV and XVI, first paragraph.

The crucial point is that the equations remain valid regardless of the outcome of the discussions on the essence of light because the vacuum hypothesis is perfectly compatible with the geometrical representation of movement. By geometrical movement is meant in this case the transformations of space in itself.

4.1 Louis De Broglie’s phase waves are not electromagnetic

Let us recall that Maxwell’s equations represent movements while the interferential fringes are the fact, i.e. the “thing which interferes”. Experimentally, the fringes are said to be of equal thickness if they appear localized on the surface of iridescent bodies; in this case light intensity can be put in direct relation with the thickness of material on which it strikes. But if they are observed under Fraunhofer’s conditions they are said to be focused at infinity, meaning that they are not at all localized because there is no place localized at infinity where their source lies; consequently something is lacking to the finite in relation to light distribution. This fact can be interpreted in a manner compatible with electromagnetism as a property of light or as a property of the object-optical instrument pair or otherwise just because Maxwell didn’t write a theory of the fact, that is he didn’t represent interference fringes. But the interference fringes immateriality requirement remains a basic requisite because it brought about Maxwell’s initial choice of representing not states but movements. To keep in the empty space the hypothesis that these fringes are not substance is equivalent to denying that optical instruments function like cannons. More precisely, it is denied that something is projected into the image like a cannon ball is projected from the initial position to the expected final one – provided aim is good.

In the thesis de Broglie does not hypothesize that optical images can behave like shot cannon balls because this is contrary to all experimental evidence. Rather he hypothesizes that an energy transfer takes place because of a convective motion of emitted photons, motion which admits a statistical mechanical model but possibly without satisfying the principle of equipartition of energy. With the statistical mixture of photons he then dualistically associates
phase waves such that the instruments focus with scattering, i.e. with some blur. These phase waves represent the intensity patterns which the diffusion of photons alone doesn’t justify. The interferential fringes observed experimentally are no longer a fundamental hypothesis of the theory but are the “visible result” of a random reality to which the theory accedes. In other words a phenomenon (i.e. the interferential fringes) is not observed but the statistical frequency with which something absolutely inaccessible to the experiment occurs.

In short: the guiding waves, of which de Broglie speaks, are not equippotent to the electromagnetic ones because the former justify interference with scattering of photons while the latter do not at all explain interference; Maxwell limited himself to ascertaining this [31]. Sometimes he calls it interference and sometimes more generically diffraction.

5 Periodicity as a property of light

The intricate subject of the essence of light was not faced by de Broglie in the thesis as we remarked above [29]. For this reason we shall limit ourselves to a few considerations on its periodicity property. In mechanics when it is a matter of phenomena accessible to the experiment, i.e. reproducible in the laboratory, it is possible to verify their degree of periodicity among other ways by comparison with a periodic device having a period near the one investigated. If the periodicity of the phenomenon to be tested is not simple, the intent can be laborious. If in the long run we really can’t distinguish any periodicity in the phenomenon it is usually agreed that it is random. Then, as no evolution law can be given because of the endless becoming of the phenomenon, we have recourse to statistics.

In optics the interferential fringes as such result indeed from the comparison between two radiations as much as possible alike. This does not effectively mean that we possess a periodic reference device but that the source is the same. The experiment allows attributing neither periodicity nor fortuitousness to the phenomenon. Let us explain this point. A spatial tessellation with a Platonic solid which is a covering can be represented geometrically by means of a finite group of movements of space in itself. Each of these movements can be expressed by parametrizing an appropriate function of the spatial variables. These functions, written in terms of the real parameter, are commonly called periodic. Just to interpret the real parameter as time, the geometrical regular arrangement can be transferred to temporal evolution. The space-time correspondence is mathematically fixed but there is a dissymmetry between the spatial and temporal variables. Time remains the parameter of a spatial transformation. It is not deduced that the phenomenon described is periodical in the sense given to the word in mechanics. Therefore this description does not make physical hypotheses regarding the nature of the fringes. On the contrary, a theoretical formulation assigning a period to a mathematical function of random variables does not specify the nature of the phenomenon but is contradictory.
5.1 James Clerk Maxwell doesn’t attribute basic colors to light

Among those who experimented and reflected on optics there is notoriously Newton \[32\]. He attributed the colors of the rainbow to light and compared them with the diatonic musical scale, tracing back sounds and light to states of motion. His reasoning is subtle; here we can quote a definition as an example: “The homogeneal Light and Rays which appear red, or rather make Objects appear so, I call Rubrifick or Red-making; those which make Objects appear yellow, green, blue, and violet, I call Yellow-making, Green-making, Blue-making, Violet-making, and so for the rest. And if at any time I speak of Light and Rays as coloured or endued with Colours, I would be understood to speak not philosophically and properly, but grossly, and accordingly to such Conceptions as vulgar Pepople in seeing all these Experiments would be apt to frame. For the Rays to speak properly are not coloured. In them there is nothing else than a certain Power and Disposition to stir up a Sensation of this or that Colour. For as Sound in a Bell or musical String, or other sounding Body, is nothing but a trembling Motion, and in the Air nothing but that Motion propagated from the Object, and in the Sensorium 'tis a Sense of that Motion under the Form of Sound; so colours in the Object are nothing but a Disposition to reflect this or that sort of Rays more copiously than the rest; in the Rays they are nothing but their Dispositions to propagate this or that Motion into the Sensorium, and in the Sensorium they are Sensations of those Motions under the Forms of Colours.”

Hereafter we shall prefer to refer to colors rather than frequencies. Newton concluded that sunlight is composed of single spectral colors from the fact that it is dispersed by a glass prism in the colors of the rainbow. Since glass technology hadn’t the interest it later acquired, he ignored that dispersion depends on the chemical composition of the glass.

Maxwell on the other hand was in a position to observe that, if that which has the property of color is light but there are pigments which radiate only light with well-defined coloring, then matter also has the property of color. In the case of rarefied gases excited electrically, color radiates homogeneously and isotropically through space, i.e. patterns are not formed. If we manage to obtain persistent light, which does not happen by subjecting a rarefied gas to any stresses, color is also largely independent of external circumstances. It can be asked if this characteristic allows using the phenomenon to define a physical time relatively to duration or periodicity. The periodicity introduced here is not in relation with that of the last paragraph; rather it should be traced back to the stability of the feeding device.

It seems that Maxwell was not entirely a stranger to the problems linked to flow of time, timekeepers and movement. In the last pages of the Theory of Heat he asks if, admitted that matter consists of molecules, the latter can be harmonised with the “light unison”. For sure there he doesn’t link resonance to a time arrow. However that may be, in electromagnetism to assume the existence of fundamental components of a particular frequency is the equivalent
of raising the method of resolution by expansion into autofunctions to a principle of physics.

On the contrary, if a physical meaning is not attributed to said mathematical method, it can be replaced to all effects with the integral transform method.

5.2 Electromagnetism is not a spectral theory

“Electromagnetic waves” thought of as propagating “monochromatic” solutions of the wave equation exist subject to foundation of a spectral theory. As long as in electromagnetism movement is attributed to ether, dynamics supplies the laws of motion of ether and perhaps explains the color of light. If movement is geometrical the only laws, those of transformation, are supplied by geometry.

A spectral principle for explaining interference and diffraction comes into conflict with Euclidean geometry adopted in particular by Maxwell. Indeed, in assuming the existence of basic physical components it is imagined that they appear progressively or suddenly with increase in resolution. If the principle is effective it attributes absolute magnitudes to the basic constituents. But Euclidean geometry does not allow introducing absolute magnitudes. To explain, if by using a telescope with better image definition we can no longer distinguish canals on Mars, the conclusion that these were artifacts at the previous degree of resolution is in agreement with the Euclidean principle of similitude. The alternative of considering the canals as real structures visible only by special enlargements is in contrast with the above mentioned principle. In this sense the geometrical theories unified with the spectral ones are contradictory.

Below in this work we shall no longer attribute several spectral components to light. But this choice involves only the theory since all optical instruments are dispersive. It must only be remembered that some disperse very little, and others – interferometers and spectral analyzers – disperse more.

5.3 Interference in optical geometry

Traditionally, taking on the Huygens-Fresnel principle, optics manuals inaugurate a new chapter dedicated to physical or undulatory optics. But this formulation must not be misunderstood. No manual intends to answer the question whether it is necessary to introduce this new basic principle to explain the phenomenology of the new chapter. To judge whether geometry suffices or whether the observations require development of a spectral theory is the concern of one proposing to write one coherent theory. Observations on which one might rest the first hypothesis are not lacking. For example, in 'On the Estimation of Aperture in the Microscope’ E. Abbe writes, “Let us assume that the fineness of a striation, a grating or a diatom is at the limit of instrumental resolution for [a microscope of] a given aperture […], then always and only two diffracted rays contribute to the image [observed with the eyepiece]. In clear field this would be the direct ray, and one spectrally decomposed, [the first is the zero order and the “colored one” is the first order of diffraction]. In this case the striation seems to be in clear and dark bands of equal width. But we know that
with [an objective of] greater aperture we would obtain a more complete image. In this the ratio between the widths of clear and dark bands would be much different.” The italics are ours. In this passage Abbe explains that interference is an image and not generically a pattern; in addition that it is the image of any flat grating, provided the instrumental resolution is insufficient. The result is that interference seems to be a property of light because it does not characterize the object observed although it is it’s image. But the microscope observations serve to elucidate the structure of the bodies observed and therefore it is the bodies which possess the property of being visible.

6 Ernst Abbe’s hypothesis on what moves

We saw that, according to Abbe, the interference pattern is an image. We repeat that the immaterial or ethereal image does not have existence independent of the thing (it goes without saying that, if the thing is not alight, the image depends also on illumination). But microscopic observation is an observation of light only. More exactly, it is about the kind of light which in telecommunications takes the name “signal”.

Now let us say what moves according to Abbe. If there is no other way to accede to something but to observe it with the aid of an optical instrument, it is illusory to assume that the image has the visual appearance which the thing would have if we could observe it directly. This justifies the aptitude to also consider images those figures which certainly do not reproduce in all detail the structure examined. With this qualification Abbe’s aptitude consists of assuming that the image is the linear transform of the anti-image. This linear transform limited to the signal represents the displacement of competence of optics. Analysis of the displacement totally disregards all information conveyed by the signal, hence any question concerning its interpretation. In addition Abbe experimented with when the image is formed and how it depends on the lens numerical aperture; then he appraised within what limits one may speak of geometrical transformation and in this case how to express mathematically the condition of formation of the image. What he called “condition of aplanatism” is clearly an invariant of the transformation. One of his works on the subject is entitled, "über die Bedingungen des Aplanatismus der Linsensysteme" [14]. In the figure we reproduce from that work the plate in which he calculates the aplanatic distortion of a square grating. Observing this plate through the lens of the microscope he saw a grating with square meshes. Abbe also explains in what this distortion differs from spherical aberration. In effect the aberrations which meet aplanatism are practically those of Seidel.

7 Geometry as linear model of optics

Geometric modeling assumes for its applicability that relations between facts are linear. Now, saying that a cannon and a mirror don’t have the same use, we
Figure 1: Aplanatic distortion of a square grating. Two arrays of orthogonal equidistant parallels are transformed into two arrays of homocentrical hyperbolias. Aplanatic distortion may be observed looking through wide-angle lenses, in case there are no convergency errors.
don’t bring any evidence that the phenomenology associated with light manifestations of matter is easier to handle mathematically than that associated with combustion, friction or viscoelastic deformations. On the contrary, we foresee excluding from the model power applications such as the surgical laser and microwave ovens.

Among the reasons for which the cannon and the mirror don’t behave in the same manner there is thus also the different use. The cannon is used for offensive purposes while the mirror is used as a detector of a signal—the one sent by the object. In signal approximation the properties required to form reflected (or refracted) images can be attributed to light just because for the purposes of the interpretation of the image the detector function can be disregarded. The image itself depends on the detection technique used (not only on the object which transmits it and the transmission channel) because it is possible to exploit a good number of different effects for detection purposes. Despite this we can take into account some instrumental characteristics in signal approximation. In this case we say that the receiver, including the detector, functions as a black box.

Below we seek to specify possible uses for geometric modeling in a theory of light in linear approximation. Geometry can take on one or the other of two functions. It can serve as a linear model of a relationship. It will be our responsibility to choose this in such a manner that the model will be linear. But this does not imply that the relationship itself is analytically a straight line in Cartesian coordinates. Otherwise we can choose to use a geometry to describe our world. Since our world is a geoid, if we intend that we represent it linearly by mapping a sphere on the plane, it will be well to clarify the meaning of linearity.

In the first case the geometric model cannot represent any peculiarity of our world. We avoid confusion if, instead of introducing space as an ineluctable modality of our perception, we immediately introduce geometry as a mathematics. In this case it is clear that the space represented supplies on the outer world the same kind of information which the cardinal number 3 supplies on the three animals Bianchina, Fiorella and Estella.

If, on the contrary we intend to accede to a physical aspect regarding the extension of the world and the access is experimental, for example an optical detecting and ranging, again to avoid confusion, it is better to specify that we are concerned with topography. Incidentally, a more traditional measurement than remote sensing is to lay a graduated ruler on the object while being careful to use a good surface plate common to the ruler and the object. Even in this case the graphic representation is conventional except that whoever uses it must succeed in orienting himself in the region represented by recognizing the elements on the map.

Abbe made the first choice. Indeed, he said that microscope images are not recognized but known. Then the linearity hypothesized for the model allows using the theory to represent the image of something. But the basic elements used to represent it are basic only geometrically. Linearity allows making use of the theory to interpret the image represented but it cannot consider it a
linear approximation of the world we live in because there is no way to compare the image with the object. We make the first choice even when we propose a geometry as a diagram and a movement as a relationship. This is Maxwell’s choice when vacuum is empty.

7.1 A proposal of Felix Klein

In 1901 F. Klein (No. LXXII in vol. II of the collection of scientific works; the article is quoted by M. Born and E. Wolf in “Principles of Optics”, ed. 1991) studied the geometrical conditions under which the image appears similar to the original neglecting higher order differential corrections. This is the range of validity of the theory of Hamilton. He concluded that the theory is inapplicable to both the \textit{microscope} and the \textit{telescope}. He believes that the limitation on the applicability of geometry might be overcome by using projective instead of affine transformations; but he warns that projective optics would give rise to a theory completely different from Hamilton’s. He says, “The impression that both \[the \text{geometries}\] allow reasoning connected to each other is recalled only by the marginal circumstance that linear relation appear at the end”. This author does not put forward projective geometry as a new typography but has in mind an application to physics of the mathematics which he himself developed.

Now we shall say what we want to show. At least one geometry, the projective one, doesn’t require specifying whether the geometrical element chosen is a point or a plane. The reason is that all the projective theorems are dualizable by substituting the element “plane” for “point” and vice versa. Incidentally, to speak of projective geometry as a geometry like Euclidean geometry or one of the infinite non-Euclidean geometries, i.e. with fixed curvature radius, is not quite correct. Indeed, projective geometry is not categorical, it is not just one geometry but many geometries, in fact infinitely many. Since the geometry chosen does not distinguish the geometrical point element from the geometrical plane element it is necessary that not even the physical interpretation make such a distinction. We can agree that the point represents the object, let’s say in the specific sense of being the stigmatic image of the point, and the plane represents the associated wave, let us say in the sense of being Fraunhofer’s diffraction pattern of the point. The above convention is equivalent to that for which the same uniform illumination characterizes both a disc and a luminous sphere; under certain conditions it is experimentally verified and under others it is not. We discuss this in greater detail below. For the time being we observe that this interpretation of wave-corpuscle dualism as a dual solution can be faced with the original experiments of Abbe and his interpretation of them. In addition, since said geometry admits the biratio as a basic invariant, it is not possible to introduce basic physical components. Abbe himself in ‘On the estimation of the aperture of the microscope’ takes the numerical aperture from the condition of aplanatism and shows that the expression of aplanatism is derived from a biratio. The same expression of Abbe can be found regardless of any consideration on the microscope in projective terms. This is done for example on the reference text indicated by Klein for projective geometry, ‘Die
Geometrie der Lage’ by T. Reye \[35\]. Then the projective transformations of space in itself allow a mathematical representation alternative to the one proposed by Hamilton in which relative movements or displacements in our world are associated with transformations. This representation of movement contrasts with that proposed by Maxwell only as concerns the possibility of connecting the electrical quantities with the mechanical units of measure. This will be fully clarified below in this work for the unit of length.

8 Images and their movements

According to Klein, the most general movement is represented considering the group of projective transformations of space in itself according to the Erlangen program \[36\]. In this work we shall not apply projective transformations of space in itself. Rather, we shall first describe the facts and then indicate which relationship to represent by means of mathematics.

We refer below to catoptrical systems for definiteness. The reason is that the specular surface of a mirror functions only as a screen and reveals (usually) a single image of the object, which is the one reflected.

8.1 Localized images

First let us consider a flat mirror. If it is in a room of a dwelling and we place ourselves in front of it we will not identify light rays but will see ourselves reflected in the mirror and, behind our image, a room of the same type as that which we know to exist behind us. Truly, since our world is a bit asymmetrical, in the world which appears in front of us the right is taken for the left. Although there is no clear way for believing that the world of the mirror is other than ours, someone might believe it different because inaccessible. The “other world” seems separated from ours by the impassable surface of the mirror only because the image is virtual. But any real image is physically accessible to us. Let us clarify further; the real image does not exist without the object and the instrument capable of detecting it but when it exists it is topographically locatable.

Let us discuss accessibility better. If we place ourselves before a concave spherical mirror, let’s say with a diameter of about twenty centimeters, equipped with a ball of foamed polystyrene and a pencil, we observe the extension of the image of the ball by going through it physically with the pencil. The image has extension in our world but we run through it without encountering obstacles because the mirror does not materialize it. Impenetrability can characterize bodies but not images. The same experience can be repeated on the real image refracted by a converging lens. But it is necessary to use the dioptric surface as a support for the image. Then the image visibly occupies a position with respect to the dioptric surface and the eyes settle automatically on the details observed. We know that an image can be detected by placing a flat screen of opaque or translucent material approximately in that position; the image then appears flat and more or less sharp. The second procedure is usual using dioptric systems
but unacceptable for mirrors. For the same reason light diffused by objects cannot be diaphragmed so as to be “paraxial” using catoptrics. Clearly, every time the image is real it diffuses the light which illuminates the object for all its extension as though it were the surface of the object itself.

If we want to generalize this observation by hypothesizing that even virtual images are in our world, we cannot use Euclidean geometry because this geometry does not allow representing inaccessible regions.

Now we want to make a distinction which seems important to us. That real images are formed in the world in which we are is a fact. Adopting a geometry such as to describe in the same way all locatable images is reasonable. Detecting the movement of a body is on the other hand a measurement operation. Even if we provided for representing the path with the same geometry used for representing images, they are facts, this is and remains a useful concept. In other words neither the trajectory nor the instantaneous speed with which a point travels over it are optical images. This is true apart from the priority we like to establish between concepts and things.

8.2 Movements of localizable images

In this paragraph we are not truly concerned with the optical detection of moving objects, rather our purpose is confined to examining one effect of increasing distance. As an example we can follow the movement of the point of the pencil looking in the concave mirror mentioned above, and supposed in relation to a graduation integral therewith. Qualitatively the correspondence between movement of the point and of its image is the following. While we cover the distance between us and the focus of the mirror, we see the image of the point move from the initial position in front of the mirror toward us until we no longer see it clearly because it is so close. Then we lose sight of it. Imaginarily it moves behind us more and more. When we pass through the focal point with the point and proceed towards the vertex it may happen that the image of the point enters into the visual field from the edge of the mirror as though it had traveled an elliptical trajectory; if on the other hand the alignment is good the contour of the stigmatic image forms visibly back-to-front behind the mirror. The image failed beyond the surface of the mirror and is separated from us by the specular surface. This surface, through which the point and its image finally appear to touch, is the only physical impediment to continuing of the movement of the pencil point. If the mirror is spherical, in addition to the one on the surface there is another point of our world in which object and image touch. It is metrically characteristic of the mirror and is quite localizable. If the mirror is parabolic, there is a single point where object and image touch et cetera.

If we had studied the path of withdrawal instead of the path of approach, for example on the ball, having it moved by someone behind us but in such a way as to continue to receive its image, we would have seen it proceed ever “more slowly” toward the focus as a limit point and flatten increasingly. All this is applied to the movement of the image without differing conceptually from
the kinematics of fluids. As the only device it is necessary that each observer hold immobile his point of view of the mirror. A repositioning of the mirror in relation to the observer implies indeed a change of reference, the graduated scale being supposedly integral with the mirror.

8.3 The information contained in the stigmatic image is that contained in the wavy image

This short paragraph doesn’t concern so much the image in geometrical approximation as it does analysis of the information contained. Up to this point we have introduced the stigmatic image of the nearby object not diaphragmed in diffuse light. The image is assumed to be reconstructed point by point, one plane at a time from the object’s shell. Although usually in optics focal rays are represented, geometrically the transformation is a central projection. For this reason optics texts call “collineation” the image constructed according to the principles of geometrical optics. The stigmatic image, i.e. the one reconstructed in linear approximation, is not flat if particular conditions such as those relative to focusing on photographic film are excepted. We showed a way of ascertaining whether the specular image also reproduces the spatial extension. Whoever has read about holography will not be surprised. Among the more common holograms are small reflecting plates which, when uniformly lighted, appear as photographs having parallax. This moving effect of the planes according to their depth is codified interferometrically during recording of the hologram. The procedure consists essentially of impressing on a film the pattern of interference between a reference radiation and the diffuse light from the holographed object. A complicated pattern remains impressed on the film. After development the plate is capable of regenerating the “wave” diffused by the object by modulating a striking radiation of the same type as the one used as reference during recording. It appears that in the approximation in which we suppose the pattern etched on the perfect plate, or the mirror free of aberrations, we can explain the not flat surface of the image by making use indifferently of the properties of radial propagation assumed for the light, or of the wavy characteristic attributed to radiation. The pictures are complementary but the decodable information from the signal received is the same. From the analysis made it would seem that we could really infer that a dual nature could be ascribed to light in the sense of de Broglie. But this is not so. This dual nature must be attributed to the receiver paired with the respective coder in the sense of the principle of complementarity.

Setting aside the theories of light, the signal received by the mirror can also be interpreted as though the instrument performed a central projection.

8.4 The delocalized image

In Abbe’s analysis the image of the light point or the uniformly lit plane is the extreme case. In the example of the mirror we begin to make plausible that geometry is sufficiently descriptive of what is observed. Let us assume that the
object, which now need not be symmetrical, and rather it is better that it not be, while it continues to be illuminated in a diffuse light withdraws behind us to the horizon while we, integral with the mirror, continue to receive its real image. In practice, whatever the form of the object, we shall finish by seeing a small luminous disc. The small disc, if the aperture of the mirror has a circular shape, takes the name of Airy disc. It is not the geometrical image of the object. According to geometrical optics as the object integral with its lamp withdraws along the optical axis the concave mirror forms an image of it ever smaller and nearer the focal point. But the analysis applies to objects quite far from the horizon and, for essentially mathematical reasons, certainly doesn’t extend to the case where the images are formed just in the focus. Indeed, geometrically the focus is a true point. First of all it is not obvious which coordinates to assign to the object which is projected in the focus. Then although in the present case we know that the object on the horizon is extended we must believe that all its points admit of a single point as image and not only the one intersecting the optical axis. It is a question of definition if we consider the focal point an image of something or not. If yes, the anti-image must be posed by definition. Making the Abbe-Klein hypothesis the focus is included (with its antitransform). But Einstein claimed that to do physics it is necessary to measure with the double decimeter and with the clock with escapement and balance wheel. It is clear that the distance of the horizon cannot be measured even in a Gedankenexperiment by affixing a graduated line. With the words of Klein added at the foot of no. XXX of Vol. I for the 1921 edition, “Occasionally Einstein opposed me with the following motivation: the transformation by reciprocal radii preserves the form of Maxwell’s equations but not the relationship between coordinates and values measured with rulers and clocks”.

As mentioned, with the Abbe-Klein hypothesis the focus is included. Mathematically it can be defined, “the anti-image of the focal point is a point”. Since the transformation is invertible, the anti-image also belongs to space, which topologically is simply connected.

Inclusion of the focus, just like its exclusion, can be justified physically. We observed that Abbe found himself having to interpret the images especially when comparison with the object is not possible. In that predicament he imposes that the linear anti-image of the image contains all and only information accessible on the object. Clearly when the geometric image consists only of the focal point, even if we suspect or know that the object is not dot-like, from the received signal we can infer only that it is located along the direction of the optical axis in an indefinite position.

The interpretation given can be completed both physically and geometrically according to Abbe-Klein. The line of reasoning is the following. As the object withdraws from the mirror the light which it diffuses in that direction illuminates a broader zone. Thus the edge of the mirror diaphragms it under an ever narrower solid angle. The light apparently emitted by the object is ever more collimated laterally or even more coherent spatially. If the object is sufficiently far the only light effectively received of all that diffused is the zero order of diffraction. It can be said that the aperture is too small to form an image.
In this manner the reflected image, in addition to being, in agreement with geometrical optics, always smaller and nearer the focal point, is also always less detailed because it diffuses the only light the mirror receives.

We can assume that the mirror reveals on the principle of the holographic plate. Temporarily the analogy would appear with greater clarity if there were a way to receive a reference radiation also. Let us explain this. If the light coming from afar were sufficiently intense, let us say like sunshine, and if we put ourselves in the observation conditions of Newton, we would see rings due to the presence of the reference light. In the case of the mirror, a flat surface is lacking, pressed against the convexity of the mirror and capable of reflecting partially. Consequently the appearance of the image is rather uniform and even rather extended. It doesn’t cover all of the plane because when a mirror is illuminated it also produces an image of itself, the exit pupil, to which geometrical optics can be applied in the same manner. Even if in the focal plane there can be neither more nor less light than that traversing the exit pupil, due to the place where the Airy disc is formed it is not its geometrical image in the conventional sense. Rather it has to do with a modulation of direction of the light originated by the mirror and superimposed on the light coming from afar. Fraunhofer’s observed diffraction figure is mathematically the transfer function of the mirror, considered as a detector, in the sense that is attributed to the term in electrical engineering. We can agree that the diffraction image of the point alone would uniformly illuminate a plane (Abbe calls the relative condition “aplanatism”).

The diffraction pattern of the point, thought of as a geometrical plane, contains the same information as the focal image, thought of as dot-like; even in interferential terms the object is found along the direction of the optical axis, direction which in the absence of a reference radiation is arbitrary. In a word, we can say that a spatially coherent light in the absence of other light sources illuminates a very extended zone uniformly or that the image of the point on the horizon is “delocalized” to the fullest. Geometrically the previous definition is completed thus, “The transform of the anti-image is a plane”. Clearly it is not a plane of points. The geometry compatible with the double possibility of representing the focus point of a spherical mirror is the projective one. But the projective plane is a one-sided surface. Being it nonorientable, the mathematical distinction between left and right, as well as between up and down is without a difference.

### 8.5 Experimental interpretation of the uncertainty

We have described both the “geometrical” and the “diffraction” appearance of a point on the horizon. Given a spherical mirror and nothing else it does not correspond to two different observable possibilities. On the contrary, despite the interpretation being double a single image is observed, in each case an Airy disc. This disc limits by diffraction the quality of the images produced. In the case of the mirror, if the image belongs to a physical point, let us say a star, the disc presents itself accompanied by an edging in the form of a small number of more or less pale concentric rings which recall vaguely the photograph of a
wave caused by the fall of a small object into a basin full of water. Detection of the direction light is coming from with respect to the principal optical axis of the mirror cannot be arbitrarily precise because of the modulation introduced by the finite aperture of the mirror. The spatial extension of the disc around the focal point can also be made plausible with the impossibility of exactly localizing a point on the horizon.

But the disc is also an effect of the aperture stop toward the light *diffused by every point of a nearby object.* In this form it confers a more or less granular appearance on the images [16].

It is clear that the edged or granular appearance of the images has no counterpart in any basic structure of the object and/or light, which is revealed thanks to the choice of an experimental arrangement. Rather we can say that some uncertainty either about the direction of the light or about localization of a point in the spot which represents its image troubles this kind of optical measurement.

### 8.6 Physical characterization of the objects requiring the point/plane geometrical duality

In para. 8.4 we sought to make it physically plausible that two images correspond geometrically to a dot-like object. In geometrical approximation the two elements point and plane represent the above mentioned images. We introduced the “object point” as the anti-image of the image of a material object extended and illuminated with diffused light but on the horizon. We said that under these conditions the object is such that the mirror cannot magnify it. Since a plane is extended, we must clarify what is intended by “magnification”. With this word Abbe designates the *increase in discernable structural detail.* This means that the solid angle under which we see the object is in relationship with the information on the detail. Neither a structureless ball nor a homogeneous plate would ever be magnified according to Abbe. Vice versa, if relative position/direction are the only accessible information, any object behaves optically like a ball.

Perhaps it is worthwhile clarifying how the microscope lens functions according to Abbe using an analogy. Let us consider a radio signal. Everybody knows that the useful information is contained in the modulation while the carrier serves to allow tuning on the channel. Abbe presents the function of lens aperture exactly in these terms. He writes that the fineness of detail in the image does not depend on the luminousness of the collimated beam but on the modest percentage of light diffracted by the sample (*über die Grenzen der geometrischen Optik.* p.173). It is the magnification so understood that requires large lens numerical apertures and possibly homogeneous immersion lenses, typically in cedar oil. Returning to us, it is possible that Abbe didn’t believe an unenlargeable object microscopic. But it seems to us that we can identify the unenlargeable object with the one with which de Broglie associates a dual nature.
Figure 2: Geometrical construction of the real image for spherical mirrors according to Newton. ACP is the cross-section of the mirror with center E and vertex C; QC is the principal axis; t is the tangent at P. Q is the object-point, q the image-point. The angle of incidence of the ray QP, EPQ, and the reflection one, qPE satisfy the reflection law. Despite the specific ray QP not being paraxial Newton’s law still holds.

9 De Broglie’s hypothesis in the context of electromagnetism

If the focal point and its antitransform belong to the domain of transformation, to the dual anti-image-image transformation corresponds a shifting of the dot-like object from where it is to the focus of the mirror. We shall see below when the shifting indicated is a movement.

Let us assume first of all that the object is continuing to withdraw from us in accordance with any dynamic law of motion. According to Newtonian dynamics the withdrawing of the object can be unlimited and in this case a trajectory of not finite length is associated with it. But if we consider the path of the point representative of the image, it tends towards a point on the focal plane as limit point. The interpretation that the only optical information we are allowing for is on the relative direction of the movement of the object corresponds to this behavior of the image. But, due to the invertibility of the optical path, the path of the image is such for the object also. In other words, the geometrical support is identical for the path of the image and for that of the object. Let us assume now that reciprocally the object travels from the center of the mirror through the main focus point towards the vertex. This is a movement so that the support cannot have discontinuities in a neighborhood of the focal point. In fact the propagation trough the focus is not opposed by
anything. As we noted above, when the mirror is spherical the representative points of the object and image on the path coincide in the center and vertex of the mirror. Then geometrically the unlimited withdrawing of the image must take place along a finite path and the common support of the two trajectories is homeomorphic with a circle. Experimentally it is very elusive to describe what happens to the image when the object illuminated in diffuse light passes through the focal point because in this zone the mirror amplifies to the maximum, the manufacturing imperfections are much enlarged and the increase in detail surely does not correspond to the magnification. This difficulty arises because a “flux” of the image through the focus is imagined. Not even when we can consider the image similar to the original is it admissible to call the light flux laminar. So if the focal point has not been whitened by the representation, hydrodynamics is difficult to apply. In addition, if the dynamic laws of motion could be applied to the image also it would not be reasonable to assume that in an arbitrarily short time interval, as measured by a clock, this might travel from where it is behind us to behind the reflecting surface. Thus in mechanics, boundary conditions are specified on the reflecting surface, it is admitted that the image jumps and the specular reflection is not considered a movement. But the specular reflection is the effect which justifies recourse to non-mechanical formalisms. Let us recall that the trajectory is not a fact but the result of a measurement operation. This measurement can be made by putting the ruler through each pair of positions successively occupied by the representative point and taking transit time into account, but can also be done by a theodolite. Remote ranging is an optical reception as much as specular reflection, only that it is applied to objects further away from the measuring instrument than the focal distance. If mathematical formalism can differ from that adopted in mechanics, the horizon aligned with the optical axis can be represented on the geometrical support as a transformed geometrical locus of the focal point and indicated by the symbol $\infty$. With this convention the marker point of the image no longer jumps but passes through that point. Since it is apparently not possible to find on the curve of support a neighborhood of $\infty$ such that all the points are at a lesser distance therefrom than a predetermined length the trajectory is not measurable by means of a ruler along the entire path. Our present interpretation justifies Klein’s warning (recalled in para. 7.1) that projective geometry would give rise to a theory of light completely different from Hamilton’s.

The formalism foreign to the mechanical laws of motion exists already. In fact Klein shows by substitution of the analytical expression of the reflection transformation in Maxwell’s equations that movements of this kind are solutions of it. The analytical form of the transformation is termed inversion or transformation by reciprocal radii. In Chap.XI, Vol.1 of his Treatise Maxwell himself deals with the same kind of solutions. In particular, being the theory of light dual, if the movement terminates in the focus, the solution also allows for de Broglie’s hypothesis. As concerns material bodies the latter hypothesis says: “It is impossible to ascertain the mechanical structure of an afar object by optical means just because of the assumed duality of the ethereal images”.
9.1 Geometrical interpretation of Isaac Newton’s formula for spherical mirrors

We mentioned above that there has always been attributed to the constructions of which geometrical optics makes use the meaning of graphic prescriptions about a physical space. These represent the content of the formulas and describe the facts. But use of the constructions in mathematics is limited to exemplification in demonstrations and the solution of problems. There is no guarantee that a particular problem is not indeterminate or does not have infinite solutions. On the contrary, if a geometry represents a model of certain facts, the mathematical expressions of laws can rest directly on the model without the backing of more tried theories such as mechanics. In relation to the purely mathematical approach mentioned which was adopted at the end for light phenomena both by Einstein and Maxwell we show below that Newton’s Axiom VI cas.2. represents a hyperbolic projective involution. In the cited axiom Newton transfers to a graph and calculates where a luminous axial point Q is reflected by a spherical mirror with center E and vertex C in the case of paraxial beams. Obviously since the light striking the mirror cannot be collimated using a diaphragm either the source Q is directional or the beams are deprived of any physical meaning. His prescription is: i) bisect any radius of the sphere, (suppose EC) in T; ii) let the point Q be the focus of the incident rays, the point q shall be the focus of the reflected ones; iii) you take the points Q and q so that TQ, TE and Tq be continual proportionals.

With his conventions,
\[ \frac{TQ}{TE} = \frac{TE}{TqTq} \neq 0 \]

The fixed point C in the figure is practically coincident with the vertex of the mirror only for paraxial beams.

This formula expresses a Euclidean result. With reference to figure 2 the triangle QTH is rectangular at H by construction. Indeed, QH is tangent to the small circle of radius TE centered at T while TH is another radius of the same circle. But the tangents to the circumferences are orthogonal to the rays at the tangential point. The theorem of Euclid in question says that the square \((TE)^2\) on the cathetus TH has an area equivalent to the rectangle which has for sides the hypotenuse QT and projection qT of the cathetus TH on the hypotenuse. When the formula is applied to mirrors both the distance of the object point and that of the image point are evaluated by the main focus of the mirror. For finite distances with a simple passage the formula for the power of the circle is written as follows (Coxeter loc. cit. p. 78):
\[ TQ \times Tq = (TE)^2 \]

To find therefrom the transformation for reciprocal radii which Klein says it suffices to lay down,
\[ (TE)^2 = 1 \]
More recently by convention all the distances are referred to the vertex of the mirror which for paraxial rays is the point indicated here by C. E. Hecht shows in his book on optics what is the relationship between the two formulas and also specifies all the present conventions on signs. We continue to follow Newton’s convention. In figure 3 we show the relationship between the graphic construction of the image by means of light rays and the projective definition of the harmonic points on a straight line. The equation of mirrors relative to the figure is written,

\[ \frac{1}{Cq} + \frac{1}{CQ} = \frac{1}{CT} = \frac{2}{CE} \]

This, rearranged as,

\[ \frac{CE}{Cq} - 1 = 1 - \frac{CE}{CQ} \]

is the continuous harmonic proportional,

\[ \frac{CE - Cq}{Cq} = \frac{CE - CQ}{CQ} \]

from which,

\[ \frac{qE}{Cq} = \frac{EQ}{CQ} \]

With this cross ratio is associated the figure of the projective geometry which we have superimposed here on the usual graphic construction. If Q tends to the infinite, q tends to the mean point of EC, i.e. T, for a theorem relative to the mean point, without it being necessary to consider dynamic properties for light. The theorem is enunciated but in different geometries, depending on whether Q takes on the limit value or not. In this manner optics leans on one or the other geometry depending on how it interprets the phenomenology. If it interprets the diffraction image of Fraunhofer in the same manner as any other optical image, as we propose, going back to de Broglie, then the construction drawn in black causes a projective transformation.

While the full system of projective transformations either with the invariant quadrics (system of Reye) or assuming imaginary geometrical elements according to Klein is difficult to deal with geometrically, it is easy to show what the graphic appearance of the transformation involved is: polarity. We exemplify the construction in figure 4. Finally in figure 5 we show the transformation implied in the case of spherical mirrors according to Newton. It associates a double ordering with the points of a range while leaving two points united.
Figure 3: Object-point Q and image-point q as conjugate harmonic points. A quadrilateral PkQEqCMLO is shown in black on the most common graphic construction of the image. The harmonic ratio among Q, q and the united points E and C does not depend on the choice of point P, provided it lays off the line CqEQ.

Figure 4: The simplest projective correlation between a point and its plane. A hyperbolic polarity is shown in order to compare it with Newton’s axiom. Q is the marker point of the object; qq is the section of its dual image; CPE is the section of the quadric (as a locus of points) that determines the polarity.
Figure 5: The involution pertaining to spherical mirrors. The relation establishes a double ordering involving each pair of points of a range. 1', 2', 3' ... locate the object Q; 1, 2, 3 ... locate the image q. A, B are invariant points. The ordering might be induced from the quadric, as in stereographic projection.

10 Summary and Conclusions

In this work we have sought to specify a possible physical content for de Broglie’s hypothesis. A priori it could be associated with three distinct types of experimental observation, to wit,

i. comparison between spectral analysis of light and structural analysis of matter. This aspect is the one thanks to which Bragg’s experiments can be considered corresponding to those of Davisson and Germer. Despite this comparison having been most directly suggested by the hypothesis, a posteriori the physical phenomena involved are too intricate to lend themselves to explain it physically.

ii. Dualism of reception, wavy or granular signal [16]. Depending on whether the electrical/optical detector used in the experiment was interferometrically locked to the signal or was rather a counter, the signal detected is interpreted as undulatory or corpuscular. In the classical schematic example of Young’s device, if both slits are open and their image is projected correctly on a screen, delocalized interference fringes appear which the photomultiplier detects by counting. But if a single slit is open, the intensity modulation nevertheless observable on the screen and which gives rise to Fraunhofer diffraction does not determine the phase shift from the reference because the radiation which acted as reference was obscured. The photomultiplier also detects diffraction by counting.

iii. Dual solution. According to us de Broglie himself proposed a con-
ventional duality. But the mathematics he associated with it lacked whatever connection to experiment and was not coherent. By facing the problem in optics, where duality was first observed, one sees that the dual aspect of light lies in the fact, shown experimentally by Abbe, that image and diffraction pattern of microscopic objects have the same informative content. Abbe found this result by examining separately the contributions of the lens and of the eyepiece of the compound microscope. If this fact is expressed in terms of a coherent light-signal theory, then de Broglie’s hypothesis is a hypothesis of mathematical nature formulated in the framework thereof. As a theory of light-sIGNALS we have considered Maxwell’s. Instead of illustrating this theory with a mechanical analogy, we have leaned on geometry; we based on the Erlangen program to define movements. We sought to illustrate to what the basic point and plane elements of the model correspond and how they are transformed by specular reflection.

In the new context, de Broglie’s hypothesis can be reformulated as, “The same duality attributed to their images is attributed to material objects (i.e. to the antitransforms)”. This means that the theory of light, as far as the hypothesis applies, could be applied to movements of the material objects themselves.

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